Introduction to Two-Cone HO Equilibrium

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When there are more goods than factors in the Heckscher-Ohlin Model, at least two kinds of free-trade equilibrium are possible, depending on how large are the differences in relative factor endowments compared to the differences in factor intensities. If endowments are not too different, then world equilibrium prices may be such as to permit simultaneous production of all goods within a single cone of diversification. However, if endowments are too different to permit an assignment of production to countries that will fully employ all factors in all countries using the production techniques of this single cone, then world prices will instead permit production of only some goods in some countries, and other goods in others. This second kind of equilibrium may conceivably involve complete specialization in production of a single good by every country, but it seems more likely that prices will align so that there are multiple cones of diversification. This write-up will focus on the case of just two cones, as may arise if there are three goods and two factors.

One Cone versus Two

Consider the possible configurations of unit-value isoquants for three goods if their prices are chosen at random. Figure 1 shows the possibilities. They are distinguished by whether or not the three unit-value isoquants align so that they are tangent to a single straight line. In Case 1, they do, and the intercepts of that straight line therefore indicate

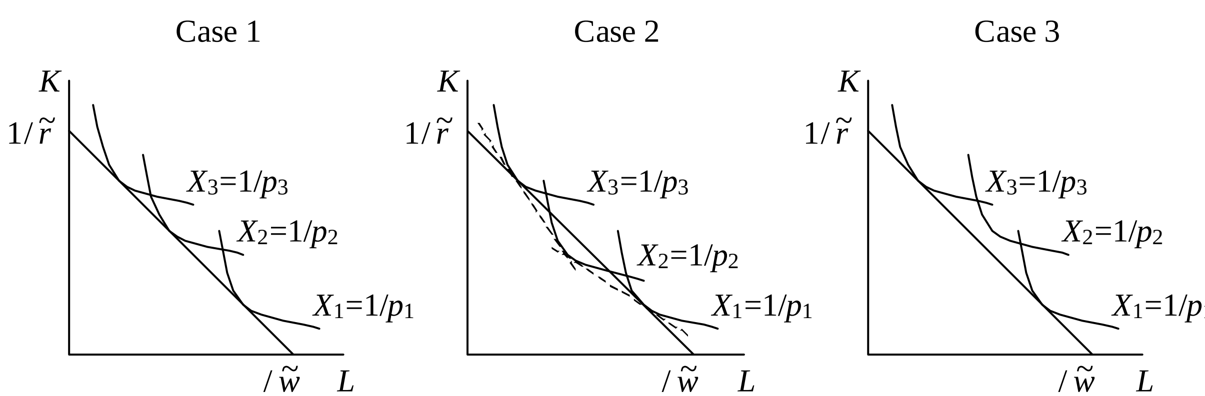

Figure 1

the rates of wage and rental, $\tilde{w}$ and $\tilde{r}$, consistent with exactly breaking even in production of all three goods. In Cases 2 and 3 they do not, but in different ways.

In Case 2, the price of the middle good (middle in terms of relative factor intensities) is somewhat higher than in Case 1, so that at factor prices $\tilde{w}$ and $\tilde{r}$, producers of X_2 would make a profit. This is inconsistent with competitive equilibrium, but in this case other factor prices exist that are consistent with zero profits in goods 1 and 2, and in goods 2 and 3, separately. These are suggested by the two dashed common tangents to these pairs of isoquants in the Figure for Case 2. Note that if either of these is a unit isocost line, then the good that is not tangent to it will not be produced in the country. That is OK, though, as long as it is produced somewhere else, presumably in another country for which the other common tangent represents factor prices.

Case 3, however, is not possible. Here the price of X_2 is lower than in Case 1, and this means that it will not be produced at all at factor prices $\tilde{w}$ and $\tilde{r}$. Nor is it possible to find other factor prices at which it will be produced, for any isocost line that is tangent to its isoquant will pass strictly above one or both of the other isoquants, indicating a positive profit and thus disequilibrium. Therefore, at the prices shown in Case 3, production of good 2 is not possible anywhere in the world under free trade, and these cannot be equilibrium world prices. So only Cases 1 and 2 are possible.

It follows that world prices can *not* be selected randomly in order to achieve an equilibrium. They must be related to one another somehow, so as to fall into either Case

One-Cone Equilibrium

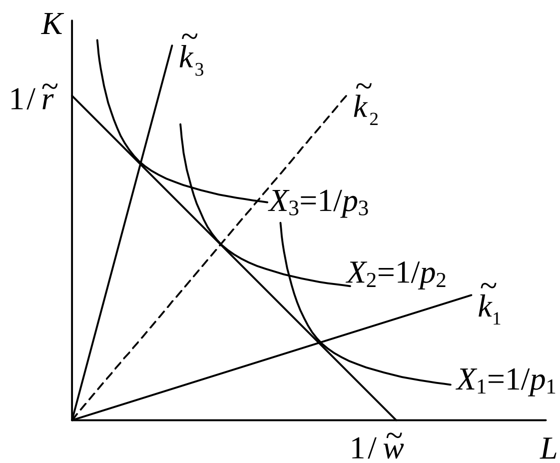

Figure 2

1 or Case 2. And while Case 1 may at first look unlikely, since it requires the three prices to align the unit-value isoquants perfectly along a straight line, it may well be that market forces will cause this to happen, just as market forces prevent the prices in Case 3.

Therefore, the two possible kinds of world price configuration under free trade in a three-good, two-factor HO model are Cases 1 and 2. This leads to the equilibria shown in Figures 2 and 3 respectively.

In Figure 2, factor prices $\tilde{w}$ and $\tilde{r}$ permit production of all three goods, using the techniques with the cost-minimizing capital labor ratios $\tilde{k}_1$, $\tilde{k}_2$ and $\tilde{k}_3$, respectively. Any country whose factor endowments lie between the most extremes of these, $\tilde{k}_1$ and $\tilde{k}_3$, will be able to fully employ its factors using these techniques. Indeed, they could do this using *only* these extremes, producing just goods 1 and 3, exactly as in the 2-good model. However, producers will also be indifferent between this option and producing some of good 2, using technique $\tilde{k}_2$. Therefore, the outputs of the three goods are not fully determined by the prices and factor endowments, but are instead indeterminate as noted and explored more fully by Melvin (1968). World outputs will be determined by world demand, but the allocation of those outputs across countries is almost certain to be indeterminate as well.

In this case, therefore, countries are able to diversify their production across all three goods for any factor endowment between, $\tilde{k}_1$ and $\tilde{k}_3$. This is therefore the single diversification cone that exists in this world-economy equilibrium. Individual countries may of course have endowments outside this cone, in which case they will produce only good 1 or good 3, with factor prices that depart from $\tilde{w}$ and $\tilde{r}$. But we might expect to find most or even all countries inside the cone, all therefore sharing $\tilde{w}$ and $\tilde{r}$ as their common factor prices. This, then, is the case of (full) factor price equalization (FPE).

Figure 3 shows the alternative case in which prices do not permit free-trade production of all three goods in any one country. At factor prices $\tilde{w}_1$ and $\tilde{r}_1$ a country

Two-Cone Equilibrium

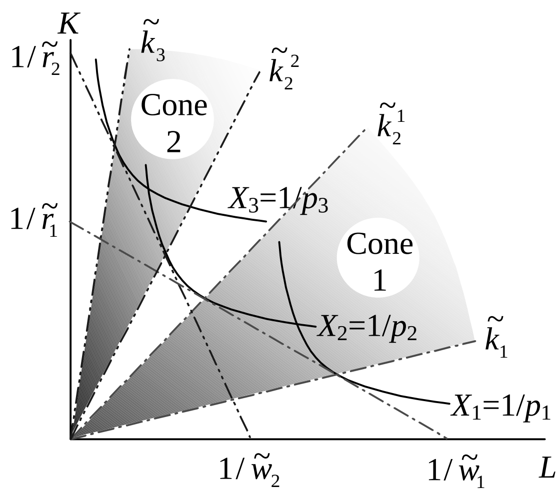

Figure 3

can produce goods 1 and 2 using capital-labor ratios $\tilde{k}_1$ and $\tilde{k}_2^1$. It will be able to fully employ its factors only if its endowment is at or between these ratios, thus in the shaded area labeled Cone 1 between them. At factor prices $\tilde{w}_2$ and $\tilde{r}_2$, similarly, a country can produce goods 2 and 3 using capital-labor ratios $\tilde{k}_2^2$ and $\tilde{k}_3$. This too is consistent with full employment only for factor endowments in the shaded Cone 2 between them. Finally, any country with factor endowment outside both of these cones, either above both, below both, or between them, will produce only one good and will have factor prices given by the slope of the corresponding isoquant at the capital-labor ratio of its endowment.

For these world prices to be an equilibrium, all goods must be produced somewhere, so there must exist countries both with factor endowments below $\tilde{k}_2^1$ and thus able to produce good 1, and other countries with factor endowments above $\tilde{k}_2^2$, and so able to produce good 3. And there must be countries, these or others, with factor endowments between $\tilde{k}_1$ and $\tilde{k}_3$, able therefore to produce good 2. A simple configuration that meets these requirements with just two countries would be to have one country in Cone 1 and another in Cone 2, although other possibilities exist even with just two countries. With many countries, one might expect them to be distributed in factor space with multiple countries in each cone, as well as others outside them, completely specialized. Countries within the same cone will share its corresponding factor prices and thus have FPE between them, even though FPE cannot hold in the two-cone equilibrium across all countries.

A natural interpretation of such an equilibrium is motivated by the difference between factor prices and factor ratios between the two cones. All countries in Cone 1 have less capital per worker than all countries in Cone 2, and they share among themselves both a lower wage and a higher rental on capital than prevail among countries in the higher Cone 2. They also employ less capital per worker in all industries where they produce than is used by the countries in Cone 2, including in the (one) industry that the two groups of countries may operate in common (good 2). All of this suggests thinking of countries in Cone 1 as developing countries, while countries in Cone 2 are developed. Indeed, the two-cone model is a natural tool for analysis of trade between the developed North and the less developed South, and it is certainly more appropriate than the more traditional one-cone version of the HO model, which without complete specialization implies identical wages across all countries, developed and developing.

Implications of the Model

Many of the implications of this model are the same as the one-cone equilibrium, since any single country can lie only in a single cone at any one time. Thus the effects of changes in factor endowments and in prices of the two goods produced, at least if the changes are small enough to leave the country in the same cone, will have effects fully analogous to the one-cone equilibrium. Thus for example, a country in Cone 1, if it accumulates more capital for a given endowment of labor, will expand it output of its more capital intensive good, good 2, and contract its output of the more labor-intensive good 1, exactly as the Rybczynski Theorem would predict. However, if such a country accumulates capital sufficiently, it will pass out of Cone 1, specialize for a time in producing only good 2, then enter Cone 2 where it begins to produce good 3 as well. Within that cone, further capital accumulation causes its output of good 2 now to fall, not in contradiction of the Rybczynski Theorem, but because good 2 is now the more labor-intensive of the two that it produces. Thus the two-cone model predicts that as a country grows through capital accumulation, its mix of outputs will change, with production of some goods first increasing and then decreasing over time, as it traverses different cones of diversification.

Similarly, a price change for a good that a country within a cone produces has the same effects on factor prices as predicted by the Stolper-Samuelson Theorem. However, as has been pointed out by Davis (1996), what matters is not the factor intensity relative to the world as a whole, but rather only relative to the other goods that a country can produce within the same cone. Thus, in particular, a rise in the world price of good 2,

Rise in p_2

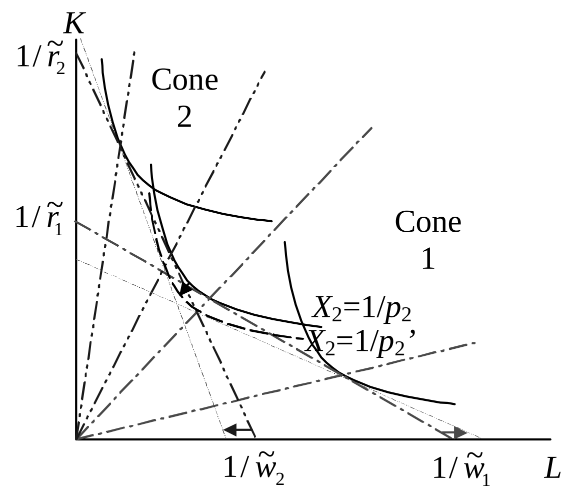

Figure 4

which is of intermediate capital intensity, will cause a fall in the relative wage in the developing countries of Cone 1, where good 2 is relatively capital intensive, but it will cause a rise in the relative wage in the developed countries of Cone 2, where it is relatively labor intensive. This is shown in Figure 4. As for the changes in the real wage, the only other complication is to consider the price of the good that a country does not produce. For the changes in Figure 4, where price of good 2 has increased with both other prices constant, this is not a problem: For countries in Cone 1, the real wage falls while the real rental rises, with opposite changes in Cone 2. For changes in prices of good 1 or good 3, the Stolper-Samuelson results apply directly for countries that produce the corresponding good, while both real factor prices fall in countries that do not produce it, since nominal factor prices are unchanged and the good costs more for consumers.

Selection of Equilibrium

What determines whether a world with free trade arrives at a one-cone or a two-cone equilibrium? The answer has been worked out by Dixit and Norman (1980), using Samuelson's (1949) device of the Integrated World Economy (IWE). The IWE refers to the hypothetical equilibrium that would obtain in the world if both goods and factors were perfectly mobile across countries. In that equilibrium, factor prices would have to be the same everywhere, and thus goods prices would have to adjust to a one-cone configuration. Exactly what those goods prices would be would depend on world demands for the goods, with consumers earning the incomes from factors employed at the common factor prices and producers producing what they demanded.

The usefulness of this device appears when one asks whether the production taking place in the IWE can be replicated once national borders confine factors to countries. If it is possible to assign IWE production of goods to countries in a way that feasibly uses exactly their factor endowments and produces the same total amount of each good as in the IWE, then that assignment constitutes an equilibrium, with FPE, of the non-integrated world economy. If, on the other hand, such an assignment is not possible, then no equilibrium is possible with FPE across all countries. In that case, at least two sets of factor prices must prevail somewhere in the world, which means, if there are multiple goods, that a two-cone equilibrium is likely.

To determine whether IWE production can be assigned to countries given their factor endowments, Dixit and Norman used a box diagram that is reminiscent of an Edgeworth box. This one, however, has the world's factor endowments defining its dimensions, and it measures factor endowments of countries from its opposite origins rather than the more usual factor employments of industries. Figure 5 shows the Dixit-Norman Box for the case of 2 factors, 2 goods, and 2 countries. The width of the box is the world's endowment of labor; its height the endowment of capital. Measuring the

2-Factor, 2-Good, 2-Country FPE

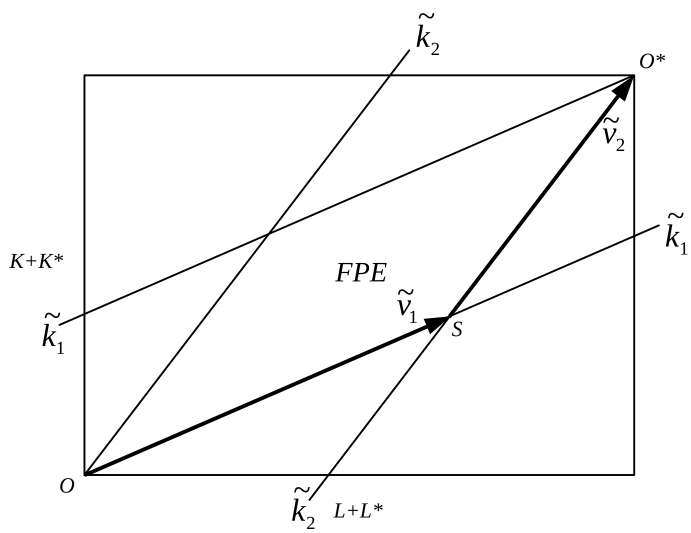

Figure 5

Home country's endowments from the lower left and Foreign's from the upper right, any point in the box represents an allocation of the world's endowments to the two countries.

Now suppose that these two countries were an integrated world economy, so that the actual locations of the factors would not matter, since they could move freely. This IWE will possess an equilibrium with certain prices of goods and factors, a certain allocation of factors to the two industries, and with corresponding outputs of the two goods equal to what is demanded in the world market at these prices. Let $\tilde{k}_1$ and $\tilde{k}_2$ be the factor ratios employed in industries 1 and 2 in the IWE. Then these also define the diversification cone, within which a country's factor endowments must lie if it is to diversify and, more importantly, if it is actually to employ the factors in these ratios. Rays with these slopes are drawn from both origins in Figure 5, and together they define the allocations of factors consistent with duplicating the IWE outputs and other variables. That is, the parallelogram formed by these four rays bounds the set of factor allocations that are consistent with FPE.

Thus, if the factor endowments of the two countries are sufficiently similar to lie within that parallelogram, then it will be possible for them to duplicate the equilibrium of the IWE, which will therefore be an equilibrium also for the trading economies with immobile factors. If, on the other hand, their factor endowments lie outside that parallelogram, meaning that the differences between the countries' endowments are larger, then it will not be possible for them to duplicate the IWE. It follows that FPE between them will be impossible. We cannot say with any certainty what sort of equilibrium will obtain instead, except that it will involve different prices of goods than the IWE, as well as specialization by at least one of the countries.

That is all we really need to know about the 2-good case, but it will help to motivate the 3-good case below to take note of one more thing. Suppose that the world's factors were allocated at point *S* in Figure 5. Then the Home country would necessarily specialize completely in good 1, while the foreign country would specialize in good 2. Therefore the heavy vector shown as $\tilde{v}_1$ must be the vector of factors used in the IWE to produce good 1, while $\tilde{v}_2$ must similarly be the vector of factors used to produce good 2.

This becomes useful when we now turn to the case of 3 goods (but still 2 factors and 2 countries). We can again imagine an IWE, which will again involve identical factor prices throughout the world. Thus the IWE prices must be consistent with a single set of factor prices for producing all three goods, exactly as in the 1-cone equilibrium of Figure 2. Factors will be employed in the ratios $\tilde{k}_1$, $\tilde{k}_2$, and $\tilde{k}_3$, and in quantities sufficient to produce exactly what is demanded of each good in the world economy. The Dixit-Norman box diagram now appears as in Figure 6. Instead of bothering at all with the diversification cones, we simply string together, one after another, the three vectors

2-Factor, 3-Good, 2-Country FPE

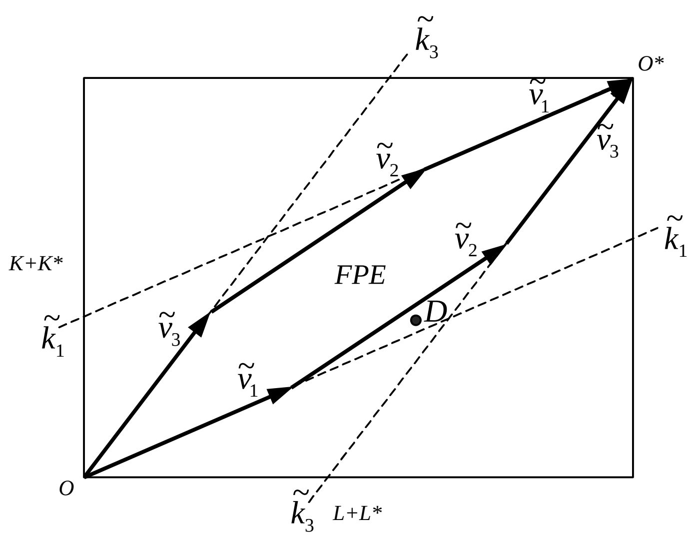

Figure 6

$\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{v}_3$ whose slopes are the corresponding $\tilde{k}_i$ and whose lengths are the quantities of factors employed in each industry in the IWE. Arranging them first in increasing order from *O* to *O**, and then also in decreasing order, they form a six-sided set of factor allocations between the countries. This is now the set of allocations for which the IWE can be replicated and FPE is possible with immobile factors; outside this set it is not.

The reason should be obvious for most of the area outside this set, where one or both countries lie outside the full diversification cone between $\tilde{k}_1$ and $\tilde{k}_3$. However, the failure of FPE for allocations like point *D*, which lies between $\tilde{k}_1$ and $\tilde{k}_3$ in both countries, is less obvious. At *D*, both countries could, if outputs did not matter, produce a mix of the three goods that would fully employ their factors. However, a little experimenting with the geometry should convince you that, in order to do so, the Home country would have to employ more than $\tilde{v}_1$ of its factors in industry 1, while the Foreign country would have to employ more than $\tilde{v}_3$ of its factors in industry 3. Thus it would be impossible to duplicate the outputs of the IWE, and markets would not clear.

This is about as far as we can go in exploring the selection of one-cone versus two-cone equilibria in the three-good case. If factor endowments are sufficiently similar to lie within the six-sided area of Figure 6, then FPE will be possible, and it will be attained. World prices will define a single diversification cone, as in Figure 2. If, on the other hand, factor endowments are less similar, enough to lie outside the six-sided area in Figure 6, then FPE will not occur. Exactly what will happen instead we cannot know for sure. Especially if one of the countries were much larger than the other – the factor allocations therefore lying near *O* or *O** – the smaller country would be unable to meet the larger's complete demand for any good. A one-cone equilibrium would again result, but with the smaller country's endowment lying outside the cone. But if country sizes are not too different, then we can expect a two-cone equilibrium of the sort studied above.

This becomes ever more likely as we increase the number of goods even further, since then even small countries can meet world demand for some small subset of these goods. With more goods, the same construction as in Figure 6 applies, but with additional vectors being strung together, so that the six-sided area becomes many sided and comes to look more like a lens. This "FPE lens," which by construction always includes the diagonal of the box where relative factor endowments are identical in the two countries, therefore defines just how different the country's factor endowments can be without interfering with FPE.[1] If factor endowment differences are larger than this, then two (or more) cones are to be expected.

[1] This is shown in Deardorff (1994), which derives the result for arbitrary numbers of factors and countries, as well as goods.

References

Davis, Donald R. 1996 “Trade Liberalization and Income Distribution,” NBER Working Paper #5693, National Bureau of Economic Research, (August).

Deardorff, Alan V. 1994 "The Possibility of Factor Price Equalization, Revisited," *Journal of International Economics* 36, pp. 167-175.

Dixit, Avinash K. and Victor Norman 1980 *Theory of International Trade*, London: Cambridge University Press.

Melvin, James R. 1968 "Production and Trade with Two Factors and Three Goods," *American Economic Review* 58, (December), pp. 1248-1268.

Samuelson, Paul A. 1949 "International Factor-Price Equalisation Once Again," *Economic Journal* 59, (June), pp. 181-197.